


# Verification of experimental saltwater intrusion interface in unconfined coastal aquifers using numerical and analytical solutions

## *Verifica dell'interfaccia tra acque dolci e acque di intrusione salina in acquiferi costieri non confinati utilizzando soluzioni numeriche e analitiche*

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### **Appendix A: statistical analysis - Appendice A: analisi statistiche**

#### TABLE - TABELLE

In the following tables:

a is the Average Significant Correlation Coefficient matrix

b is the Root Mean Square Error (RMSE) matrix (values in cm)

$h_f$  is the freshwater head

$h_s$  is the saltwater head

Tab. A-1 - Statistical analysis for the low K aquifer ( $K = 0.377$  cm/s).

Tab. A-1 - Analisi statistica per l'acquifero con bassa permeabilità ( $K = 0.377$  cm/s).

Case 1 ( $h_f = 27.7$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9990	Numerical	0	0.4448
Experimental	0.9990	1	Experimental	0.4448	0
Ghyben-Herzberg	0.9966	0.9978	Ghyben-Herzberg	2.3204	2.6772
Glover	0.9990	0.9971	Glover	1.7997	1.5170
Rumer Jr & Harleman	0.9997	0.9982	Rumer Jr & Harleman	0.4395	0.5993
Verruijt	0.9990	0.9972	Verruijt	1.4707	1.1899
Kashef	0.9996	0.9981	Kashef	0.5104	0.8758
Case 2 ( $h_f = 27.9$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9976	Numerical	0	0.3458
Experimental	0.9976	1	Experimental	0.3458	0
Ghyben-Herzberg	0.9964	0.9909	Ghyben-Herzberg	3.4004	3.4963
Glover	0.9969	0.9975	Glover	2.0272	2.0161
Rumer Jr & Harleman	0.9984	0.9978	Rumer Jr & Harleman	0.4415	0.5500
Verruijt	0.9970	0.9976	Verruijt	1.6630	1.6473
Kashef	0.9981	0.9978	Kashef	1.1155	1.1085
Case 3 ( $h_f = 28.1$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9984	Numerical	0	0.2635
Experimental	0.9984	1	Experimental	0.2635	0
Ghyben-Herzberg	0.9951	0.9936	Ghyben-Herzberg	4.4584	4.4964
Glover	0.9876	0.9850	Glover	2.1947	2.1799
Rumer Jr & Harleman	0.9908	0.9881	Rumer Jr & Harleman	0.8857	0.9560
Verruijt	0.9877	0.9852	Verruijt	1.8080	1.7998
Kashef	0.9898	0.9872	Kashef	1.7803	1.8497

Case 4 ( $h_f = 28.1$ cm and $h_s = 26.8$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9990	Numerical	0	0.4321
Experimental	0.9990	1	Experimental	0.4321	0
Ghyben-Herzberg	0.9972	0.9941	Ghyben-Herzberg	2.9165	2.6036
Glover	0.9975	0.9986	Glover	1.7715	2.1050
Rumer Jr & Harleman	0.9988	0.9992	Rumer Jr & Harleman	0.3678	0.4261
Verruijt	0.9976	0.9986	Verruijt	1.4308	1.7645
Kashef	0.9986	0.9991	Kashef	0.9547	0.6717

Case 5 ( $h_f = 28.1$ cm and $h_s = 27.1$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9993	Numerical	0	0.3544
Experimental	0.9993	1	Experimental	0.3544	0
Ghyben-Herzberg	0.9979	0.9986	Ghyben-Herzberg	1.6574	1.9193
Glover	0.9994	0.9980	Glover	0.9645	0.7780
Rumer Jr & Harleman	0.9998	0.9988	Rumer Jr & Harleman	0.2378	0.5153
Verruijt	0.9994	0.9980	Verruijt	0.6897	0.5263
Kashef	0.9998	0.9988	Kashef	0.4999	0.7941

Tab. A-2 - Statistical analysis for the high K aquifer ( $K = 0.7$  cm/s).

Tab. A-2 - Analisi statistica per l'acquifero con alta permeabilità ( $K = 0.7$  cm/s).

Case 1 ( $h_f = 27.7$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9996	Numerical	0	0.3397
Experimental	0.9996	1	Experimental	0.3397	0
Ghyben-Herzberg	0.9969	0.9965	Ghyben-Herzberg	2.1727	2.3556
Glover	0.9987	0.9984	Glover	2.2993	2.0225
Rumer Jr & Harleman	0.9996	0.9994	Rumer Jr & Harleman	0.7867	0.5069
Verruijt	0.9988	0.9985	Verruijt	1.9605	1.6894
Kashef	0.9995	0.9993	Kashef	0.2205	0.4886

Case 2 ( $h_f = 27.9$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9942	Numerical	0	0.9125
Experimental	0.9942	1	Experimental	0.9125	0
Ghyben-Herzberg	0.9961	0.9951	Ghyben-Herzberg	3.1832	3.8551
Glover	0.9977	0.9869	Glover	2.7416	2.1075
Rumer Jr & Harleman	0.9990	0.9906	Rumer Jr & Harleman	0.7249	0.8004
Verruijt	0.9978	0.9871	Verruijt	2.3570	1.7490
Kashef	0.9987	0.9897	Kashef	0.6662	1.5143

Case 3 ( $h_f = 28.1$ cm and $h_s = 26.5$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9927	Numerical	0	1.3719
Experimental	0.9927	1	Experimental	1.3719	0
Ghyben-Herzberg	0.9960	0.9131	Ghyben-Herzberg	4.1832	5.4527
Glover	0.9948	0.8685	Glover	3.5022	2.4316
Rumer Jr & Harleman	0.9969	0.8804	Rumer Jr & Harleman	0.9104	1.2516
Verruijt	0.9949	0.8691	Verruijt	3.0660	2.0193
Kashef	0.9961	0.9827	Kashef	0.9991	2.2047

Case 4 ( $h_f = 28.1$ cm and $h_s = 26.8$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9969	Numerical	0	0.6012
Experimental	0.9969	1	Experimental	0.6012	0
Ghyben-Herzberg	0.9970	0.9939	Ghyben-Herzberg	2.6650	2.9558
Glover	0.9982	0.9950	Glover	2.5502	2.1969
Rumer Jr & Harleman	0.9992	0.9965	Rumer Jr & Harleman	0.7605	0.5190
Verruijt	0.9982	0.9951	Verruijt	2.1863	1.8496
Kashef	0.9991	0.9963	Kashef	0.4324	0.9331

Case 5 ( $h_f = 28.1$ cm and $h_s = 27.1$ cm)					
a	Numerical	Experimental	b	Numerical	Experimental
Numerical	1	0.9984	Numerical	0	0.8801
Experimental	0.9984	1	Experimental	0.8801	0
Ghyben-Herzberg	0.9979	0.9976	Ghyben-Herzberg	1.4448	2.0042
Glover	0.9994	0.9971	Glover	1.6127	0.8906
Rumer Jr & Harleman	0.9998	0.9983	Rumer Jr & Harleman	0.6910	0.4946
Verruijt	0.9994	0.9971	Verruijt	1.3115	0.6444
Kashef	0.9998	0.9982	Kashef	0.1933	0.8264

Tab. A-3 - Comparison between experimental and analytical steady-state saltwater toe length (expressed in cm) for the low K aquifer ( $K = 0.377$  cm/s).

Tab. A-3 - Confronto tra risultati sperimentali e i risultati analitici in condizioni stazionarie relativamente alla distanza alla quale l'interfaccia acqua dolce-acqua salata interseca il fondo impermeabile dell'acquifero (valori in cm) per l'acquifero a bassa permeabilità ( $K = 0.377$  cm/s).

	Experimental	Lu et al. (2015)	Anderson (2021)
Case 1	28.5	34.11	28.73
Case 2	23	27.64	22.88
Case 3	18.5	23.20	19.21
Case 4	26	31.01	26.25
Case 5	40	45.98	40.89
RMSE		5.21	0.54
R <sup>2</sup>		0.9997	0.9990

Tab. A-4 - Comparison between experimental and analytical steady-state saltwater toe length for the high K aquifer ( $K = 0.7$  cm/s).

Tab. A-3 - Confronto tra risultati sperimentali e i risultati analitici in condizioni stazionarie relativamente alla distanza alla quale l'interfaccia acqua dolce-acqua salata interseca il fondo impermeabile dell'acquifero (valori in cm) per l'acquifero ad alta permeabilità ( $K = 0.7$  cm/s).

	Experimental	Lu et al. (2015)	Anderson (2021)
Case 1	29.5	34.11	27.42
Case 2	23	27.64	21.35
Case 3	18	23.20	16.62
Case 4	26.5	31.01	24.46
Case 5	40.5	45.98	38.93
RMSE		4.90	1.77
R <sup>2</sup>		0.9990	0.9994